Indian Statistical Institute

Mid-Semestral Examination 2003-2004

M.Math. I Year II Semester

Date:8.03.2004 Differential Geometry Marks: 40

1. Show that an atlas for any compact manifold requires at least two charts. [3]

2. Define $F: M(n, \mathbb{R}) \to \mathbb{R}$ by $F(A) = Determinant(A)$.

(a) Compute the derivative $DF(A)$.

(b) Show that $SO(n)$ is a regular submanifold of $O(n)$.

(c) Compute dimension of $SO(n)$. [5]

3. Find a nonconstant, smooth map between the manifolds $M = (\mathbb{R}, u^3)$ and the usual space $E = (\mathbb{R}, u)$. [3]

4. Let $M = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$, the unit sphere and $N = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}$ be the cylinder. Let $p \in M$ and p' be its orthogonal projection onto the Z-axis. Let the line joining p and p' intersect N at a point q. Show that the map $F: M \to N$ defined by $F(p) = q$ described above is a smooth map. [10]

5. Let S^n be the unit sphere in $\mathbb{R}^{n+1}$. Fix an orthogonal matrix $A \in O(n+1)$ and define $F: S^n \to S^n$ by $F(x) = Ax$. Show that F is a diffeomorphism. [6]

6. Show that the quotient map $G : S^n \to P^n(\mathbb{R})$, $G(x) := [x]$ is an immersion and a submersion, but not a diffeomorphism. [10]

7. Find integral curves to the vector field $X : \mathbb{R}^2 \to \mathbb{R}^2$, $X(x, y) = (1, 2x)$ [3]